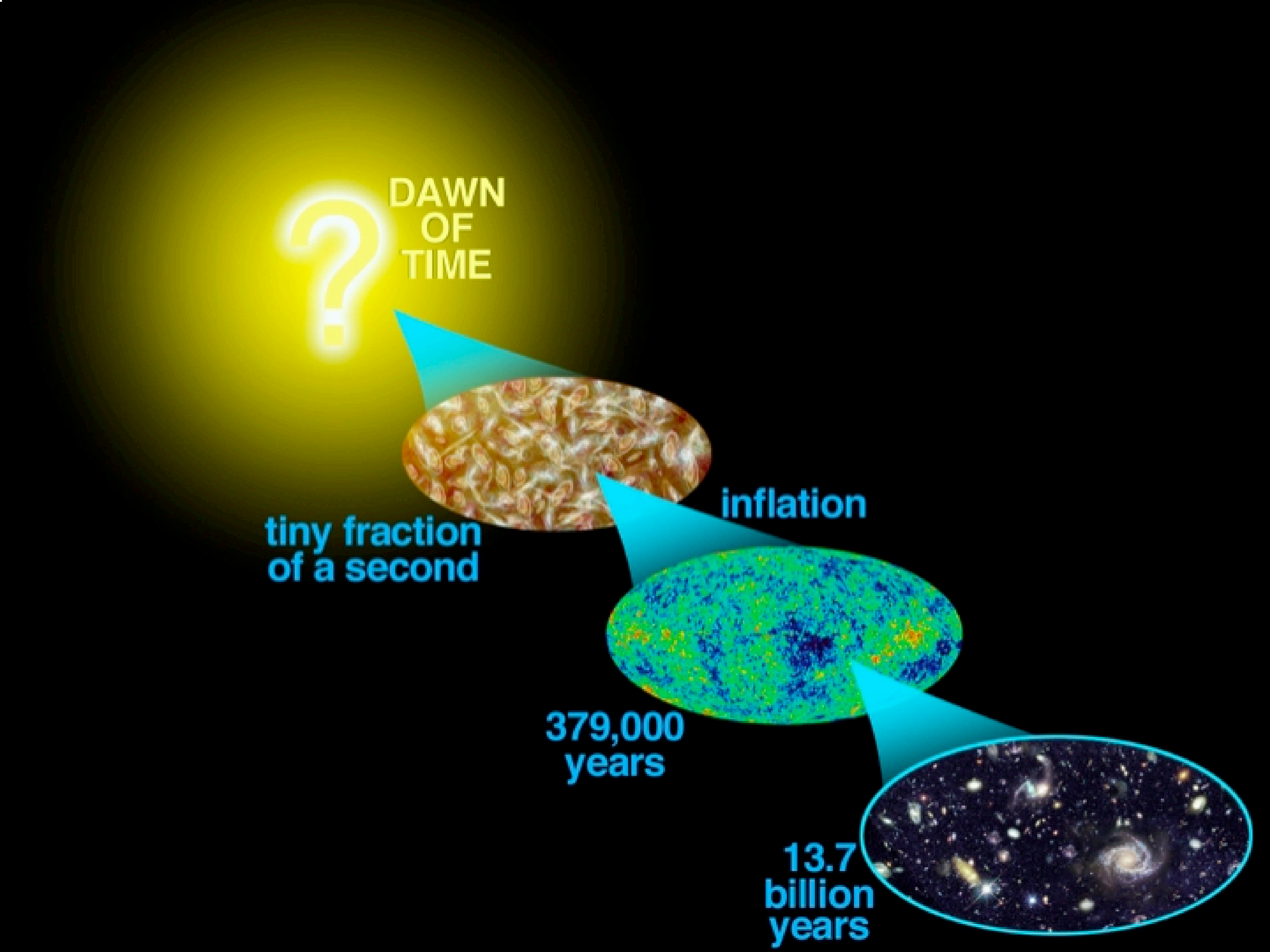
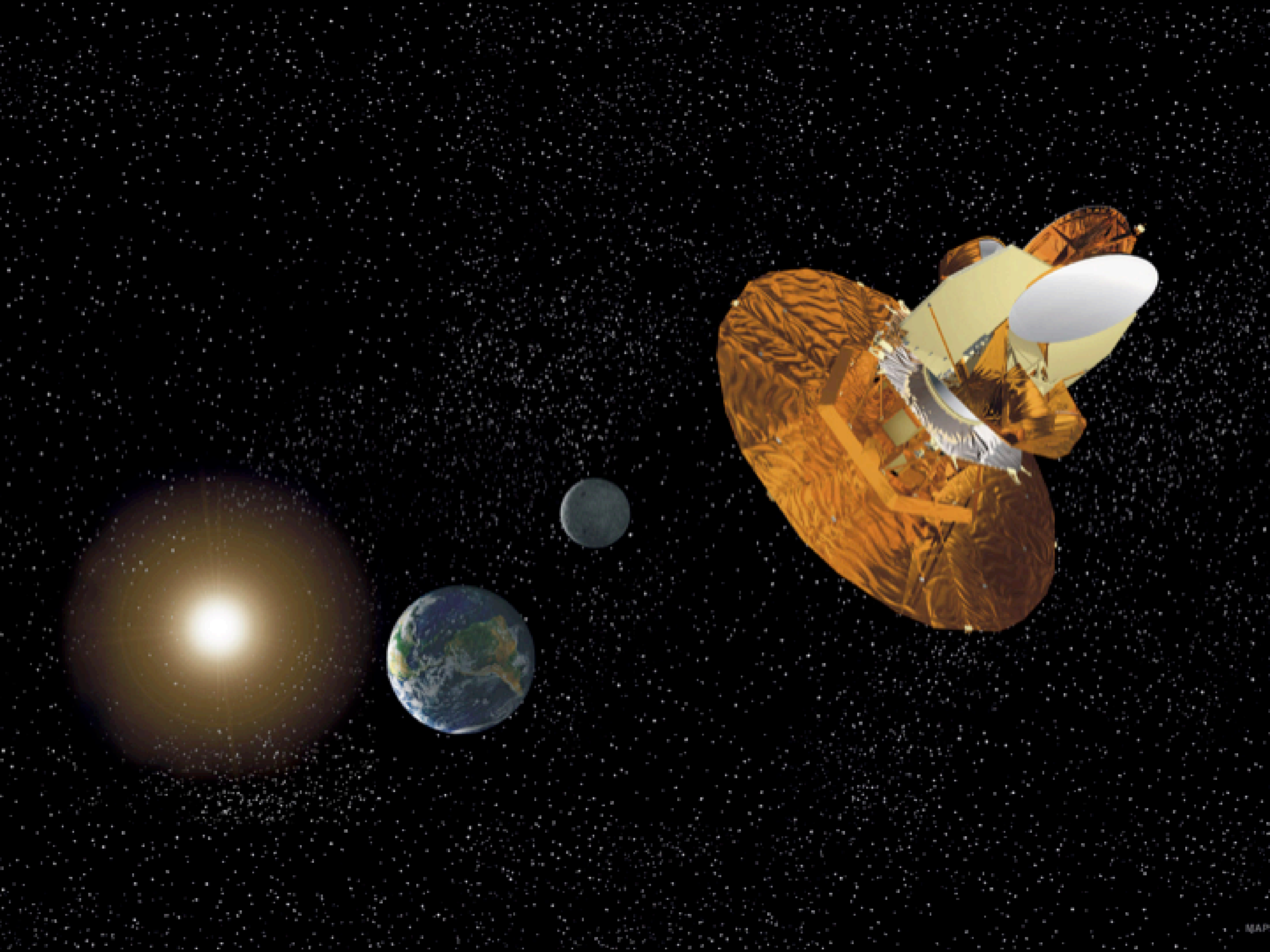
Quantum Gravity in the Microwave Sky?

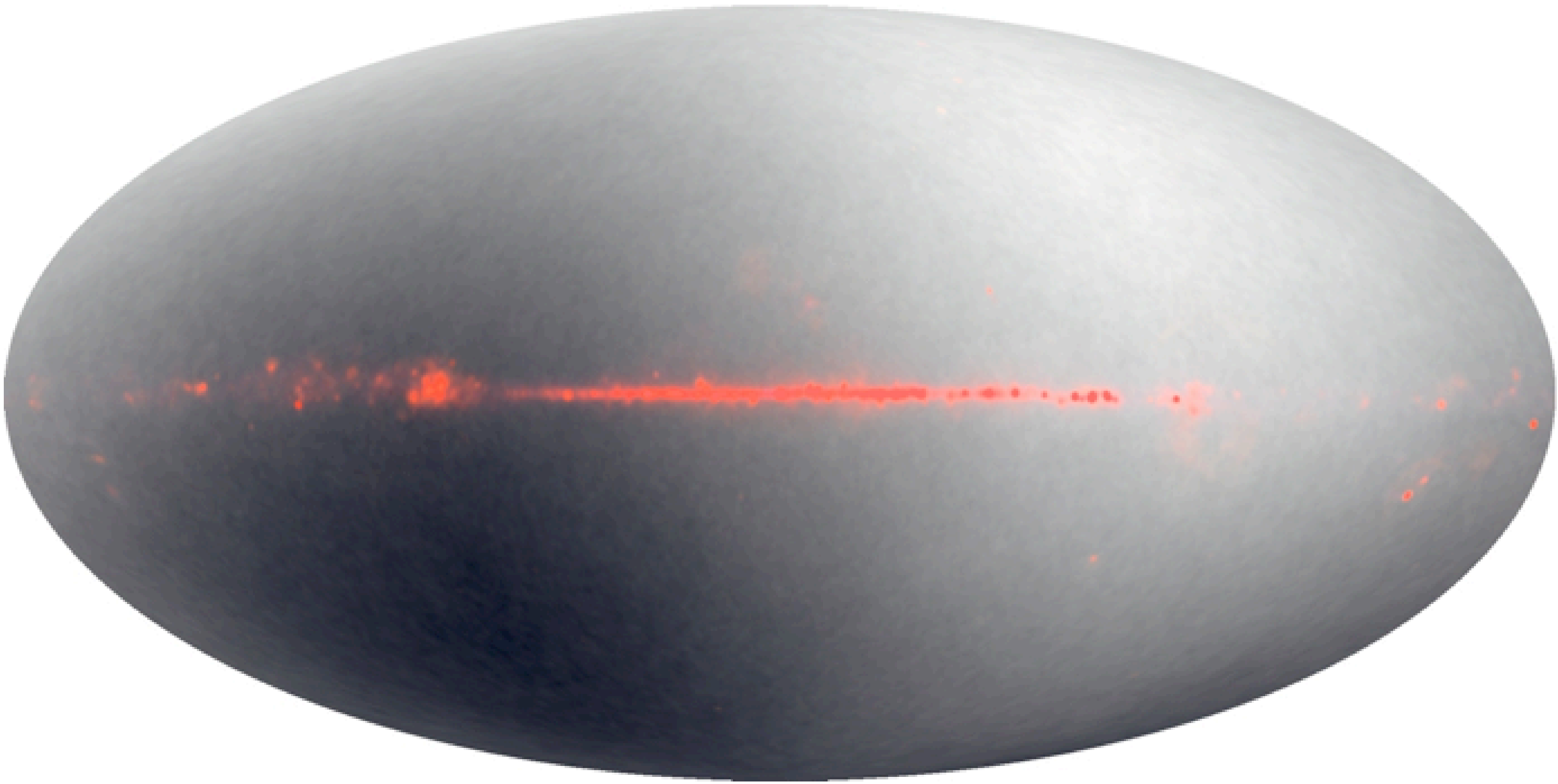
Craig Hogan, University of Washington

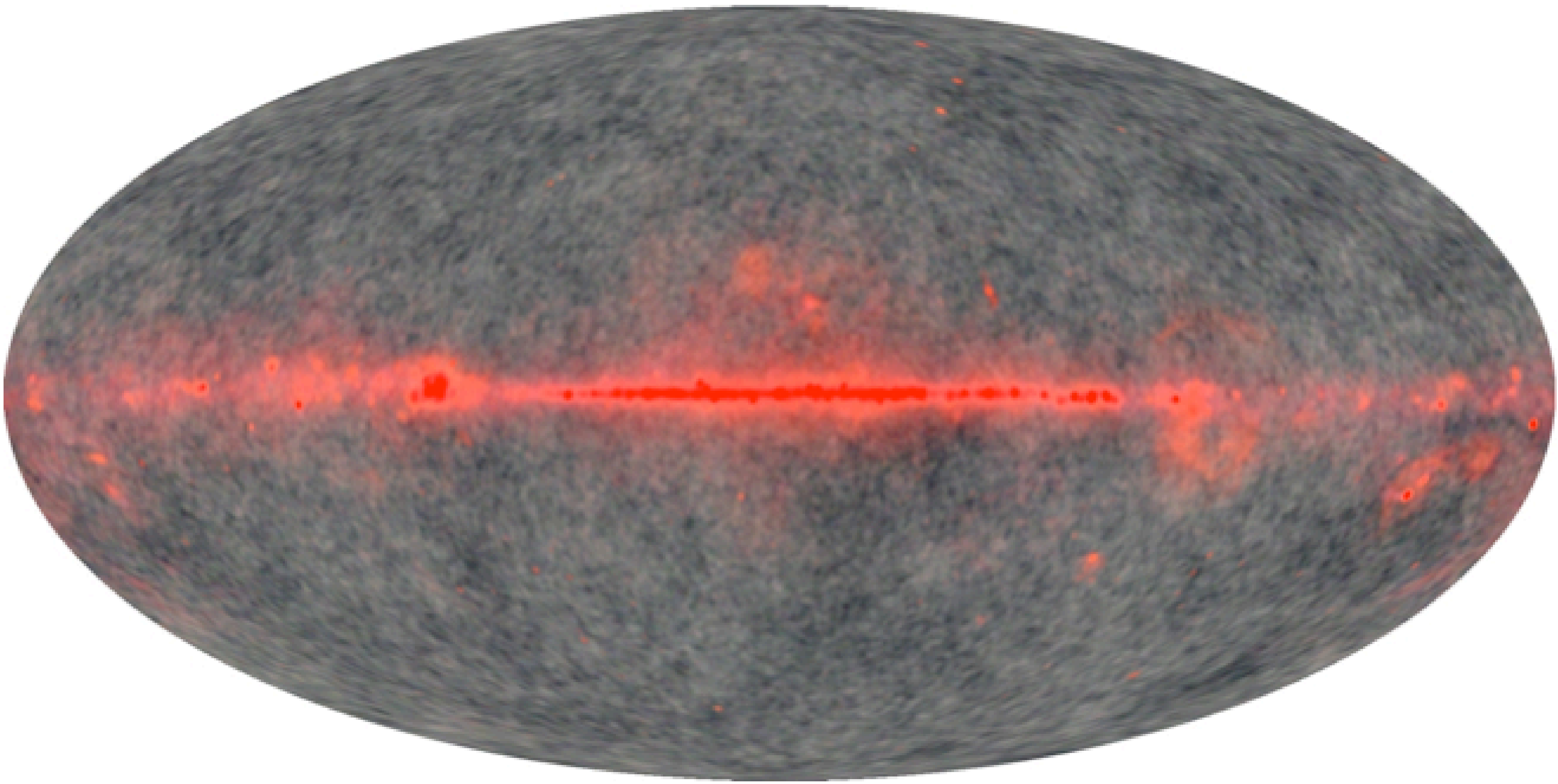
Fermilab, 9/01/04

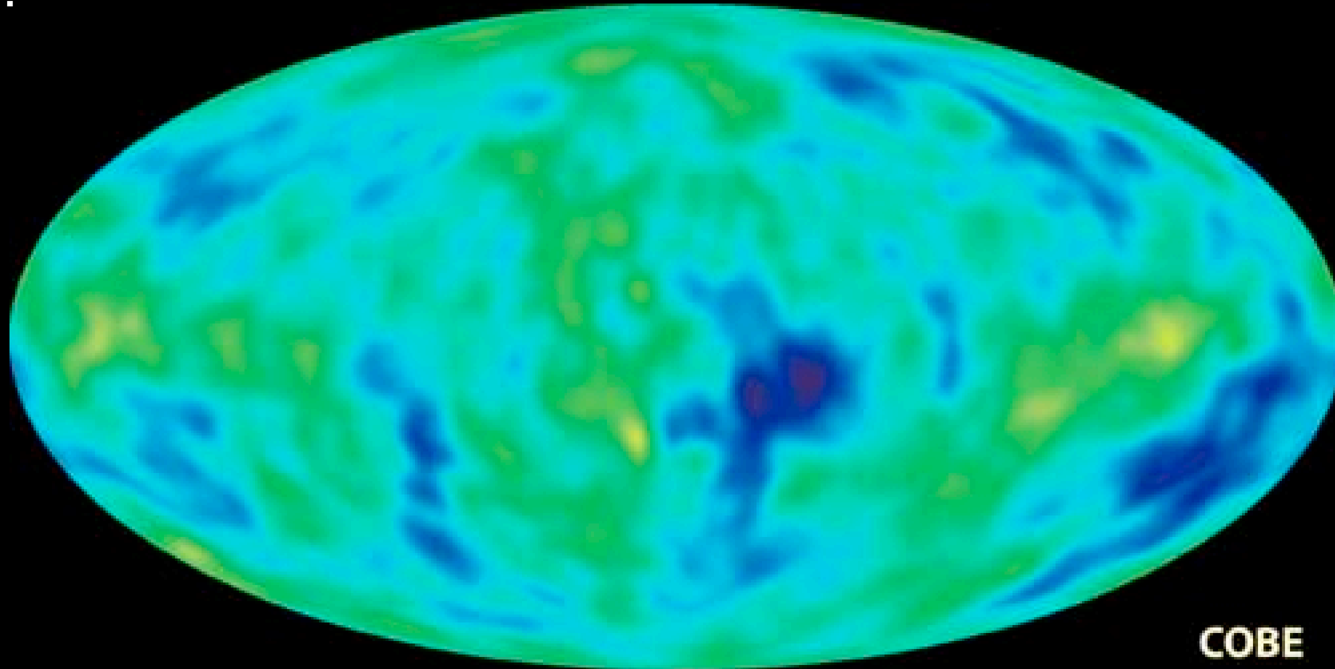
- **astro-ph/0406447, astro-ph/0310532**
- Phys.Rev. D66 (2002) 023521


DAWN
OF
TIME
inflation
tiny fraction
of a second
379,000
years
13.7
billion
years

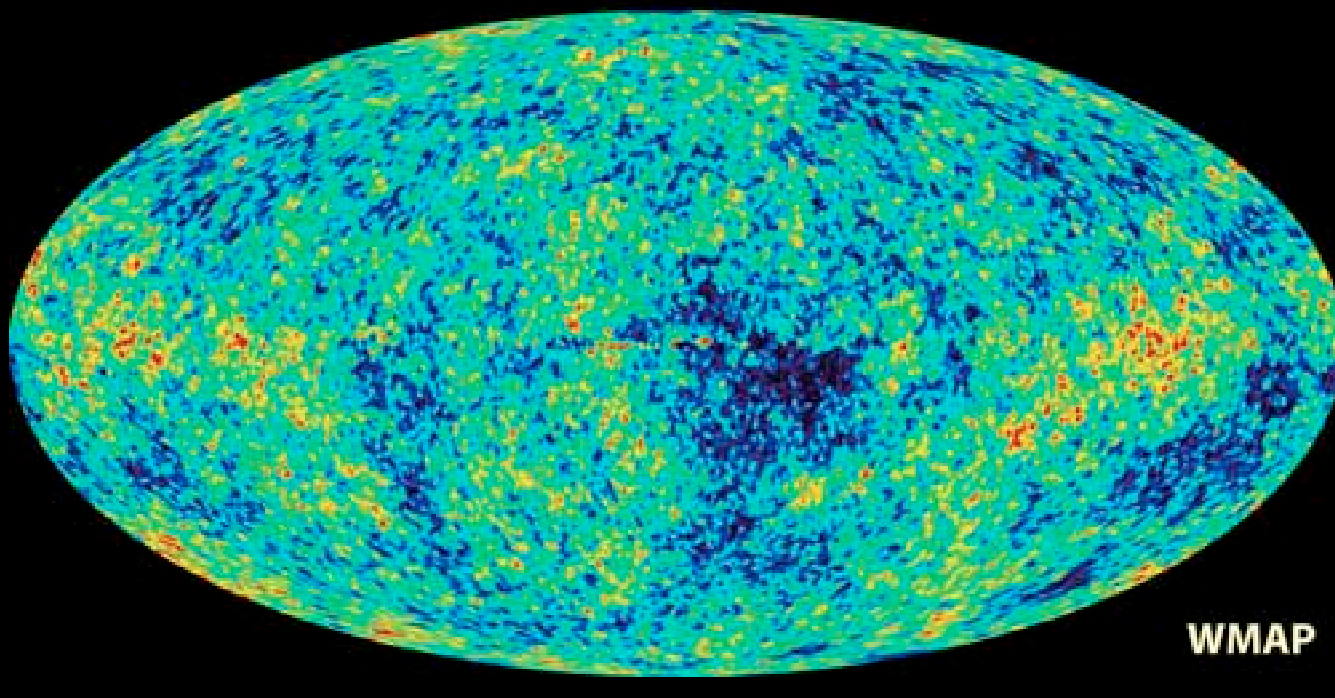




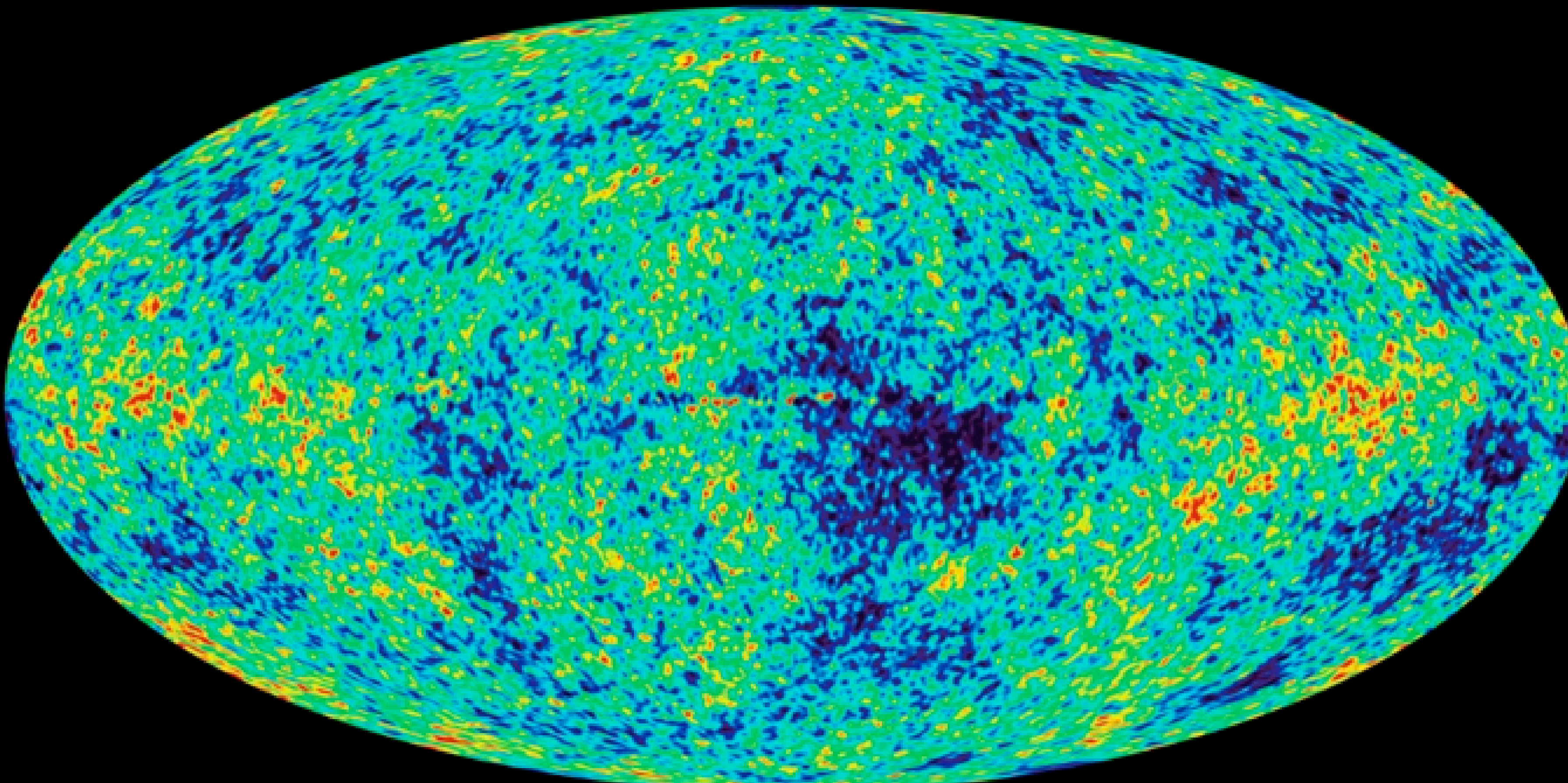


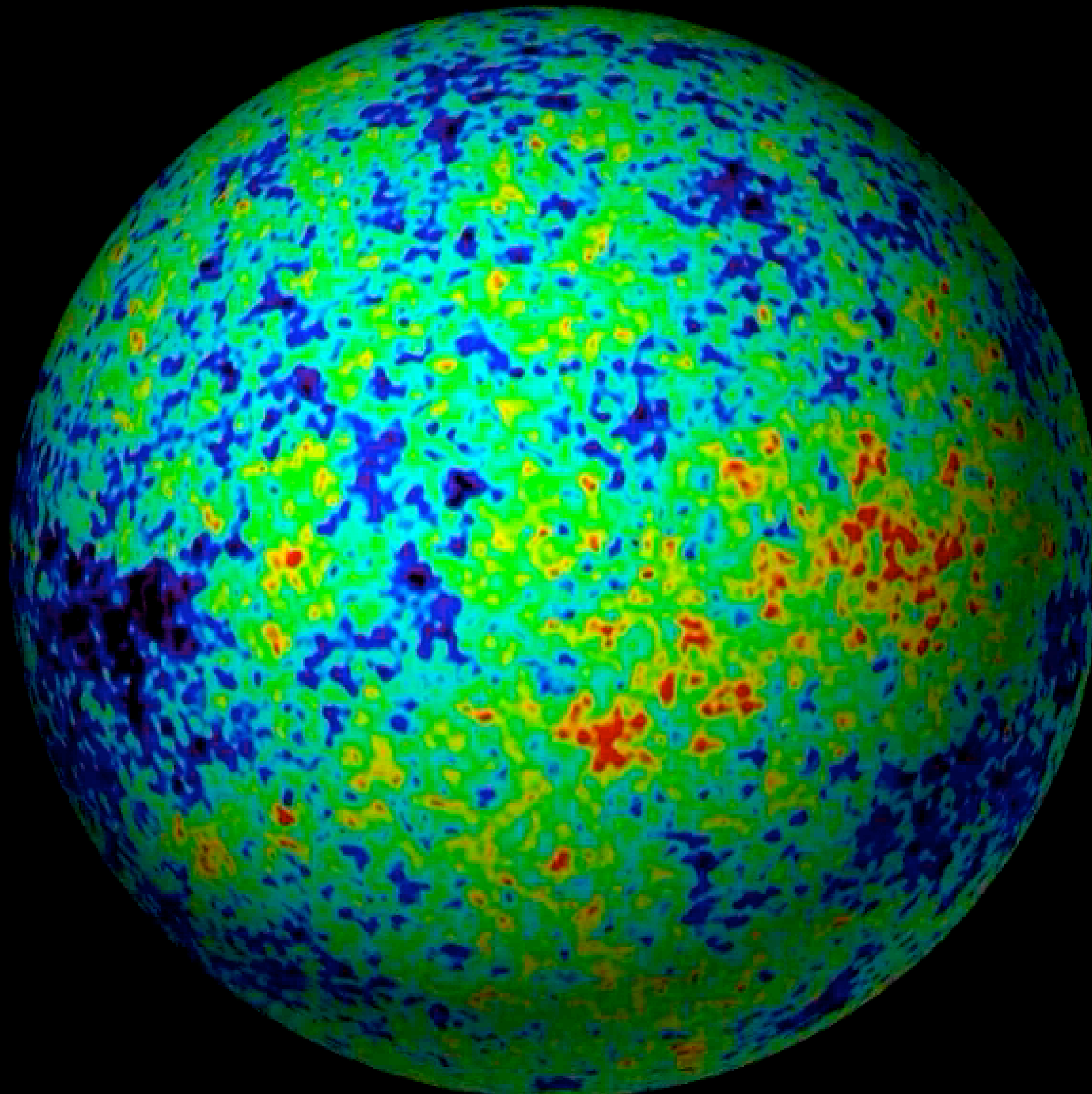


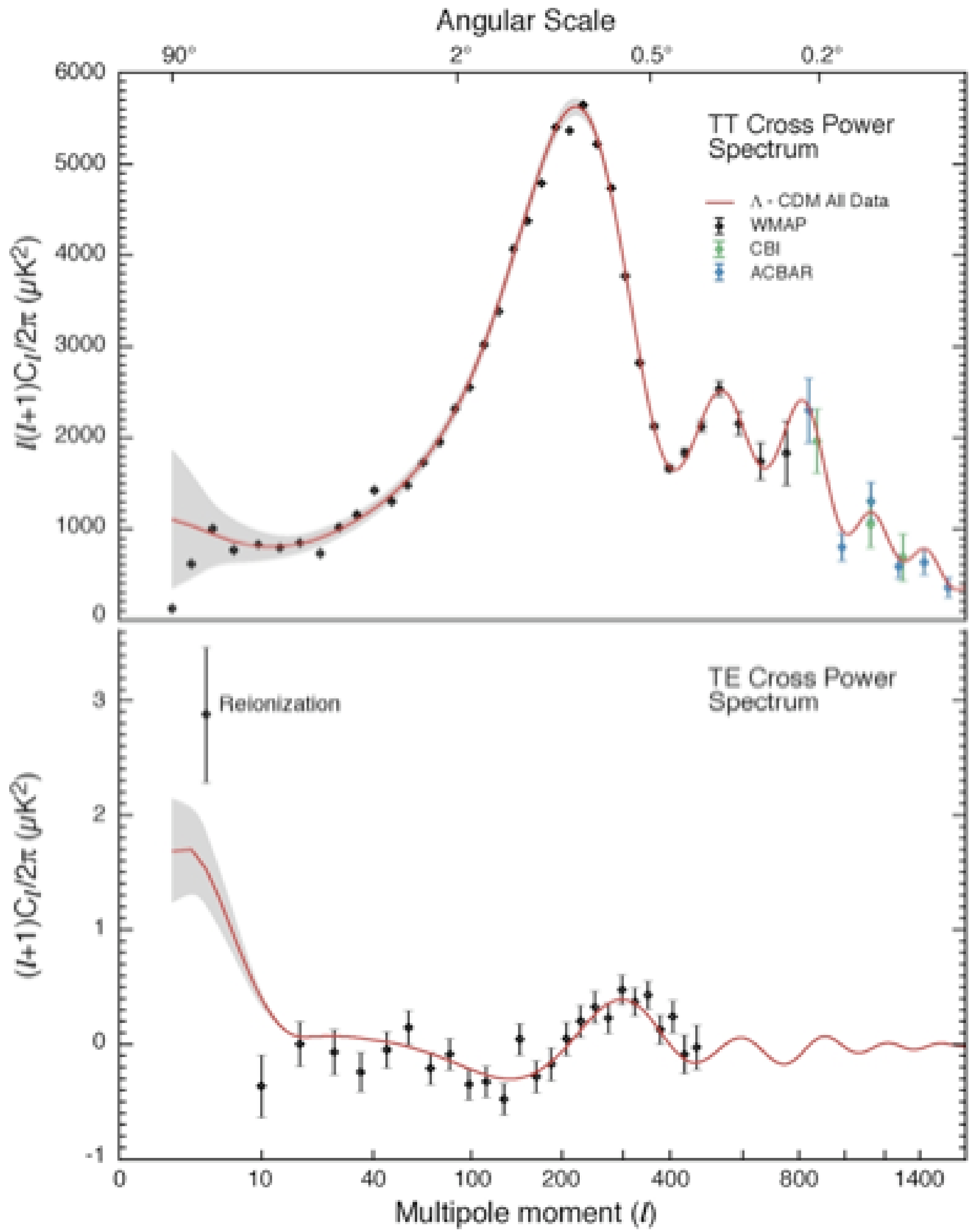
COBE



WMAP



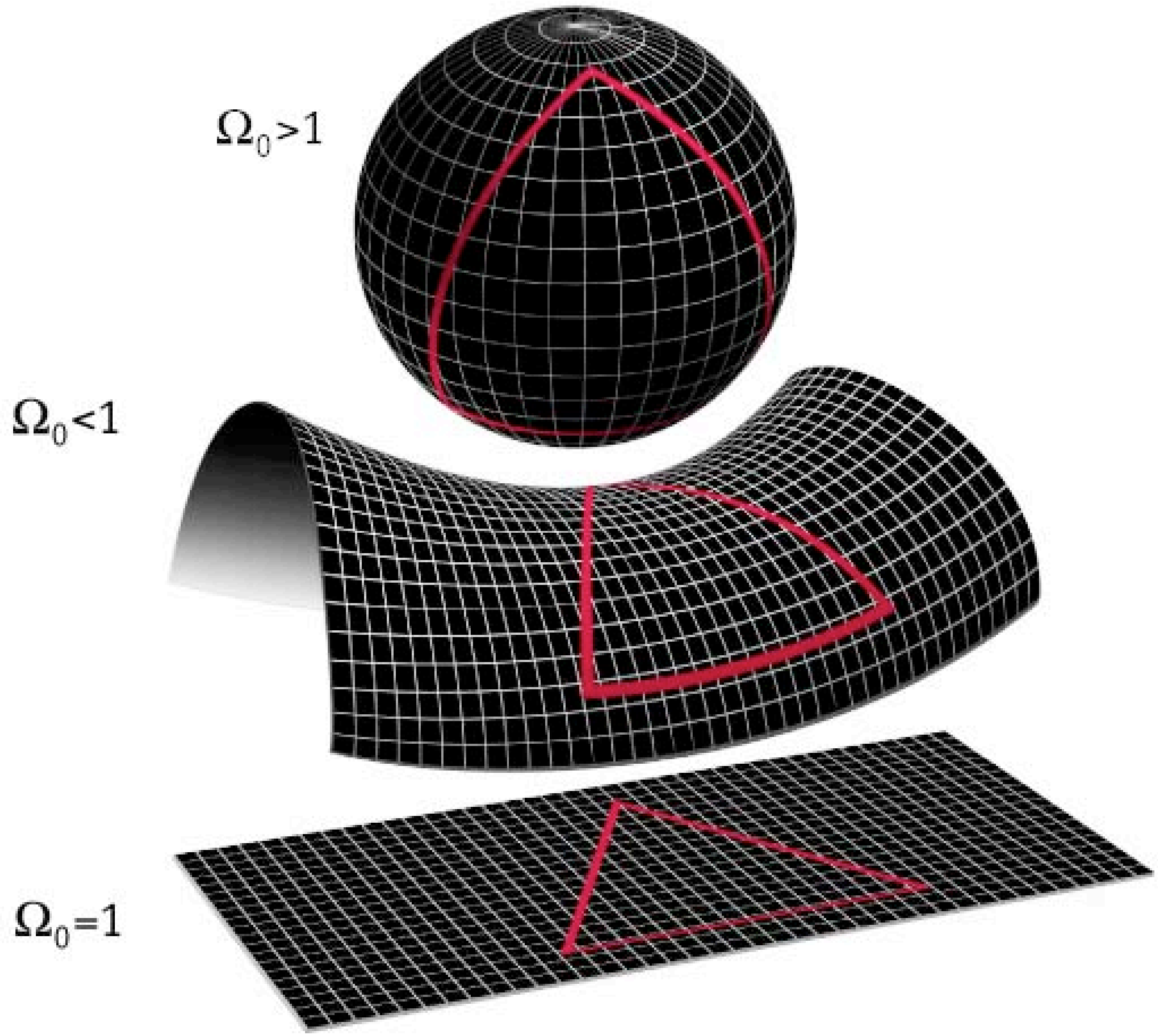




Angular Scale
90°
2°
0.5°
0.2°
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
$l(l+1)C_l/2\pi\ (\mu K^2)$
TE Cross Power Spectrum
Reionization
$(l+1)C_l/2\pi\ (\mu K^2)$
Multipole moment (l)

- The universe is 13.7 billion years old
- It is at least 100 times bigger than the part we can see even in principle

Hypersphere geometries



Quantum Origin of Perturbations

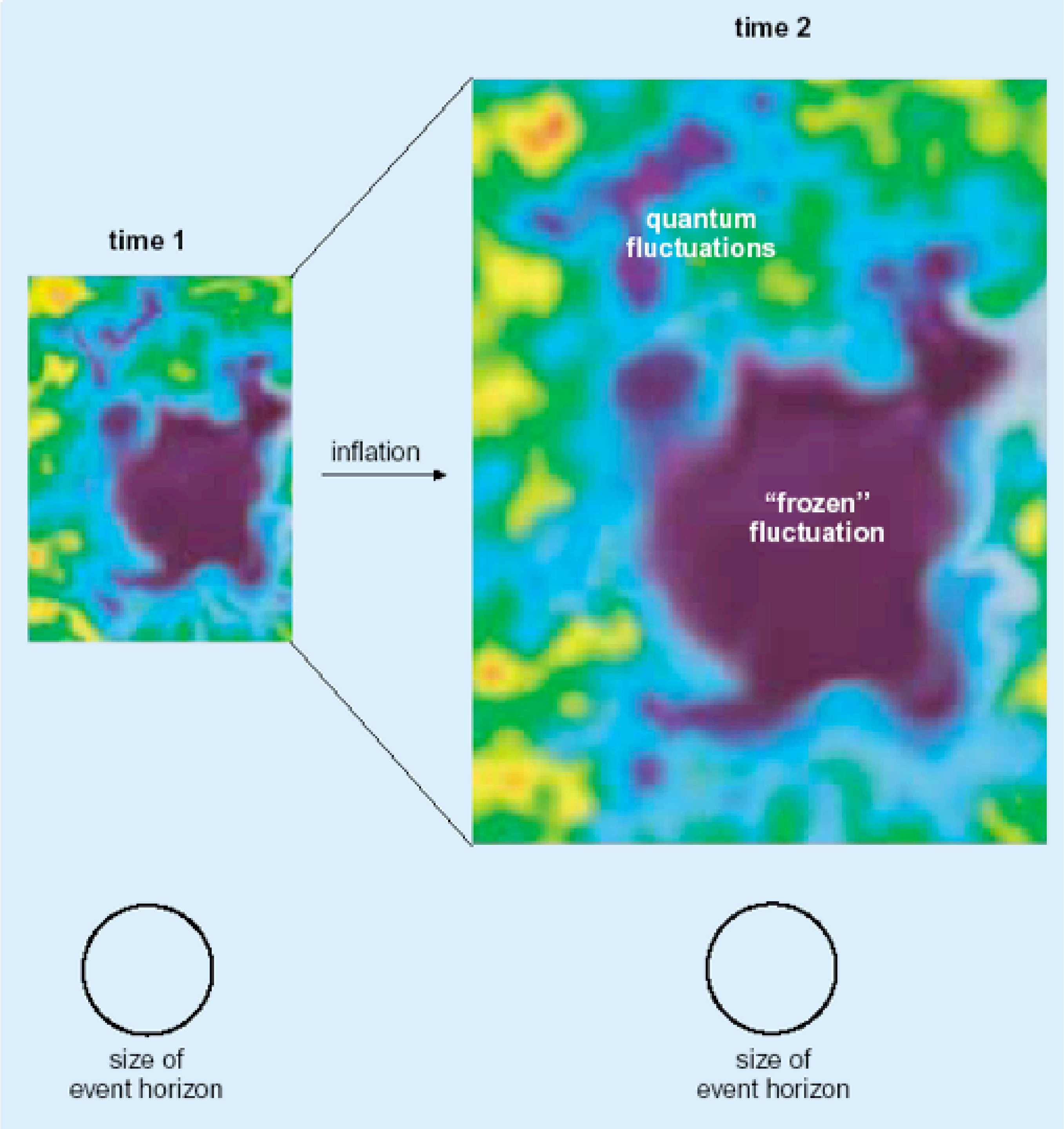
- Standard inflationary scenario: structure originates as quantum fluctuations in inflaton+graviton
- On the largest scales, we observe a direct image of primordial perturbations
- Each blob is a faithful image of about a “single quantum” during inflation

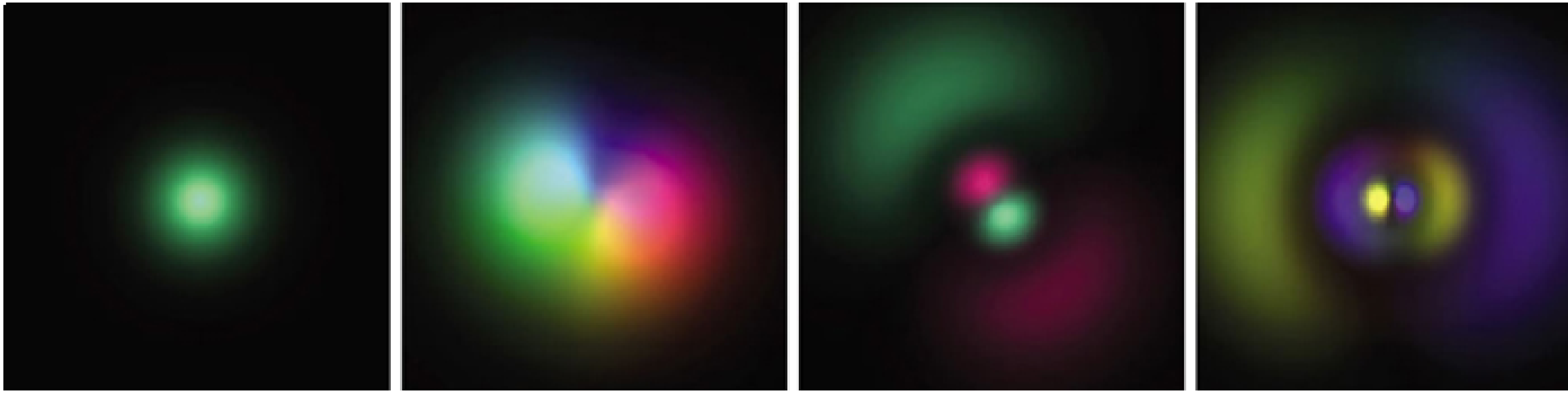
Cosmic Koan

- CBR anisotropies are both the smallest and largest imaged entities in nature

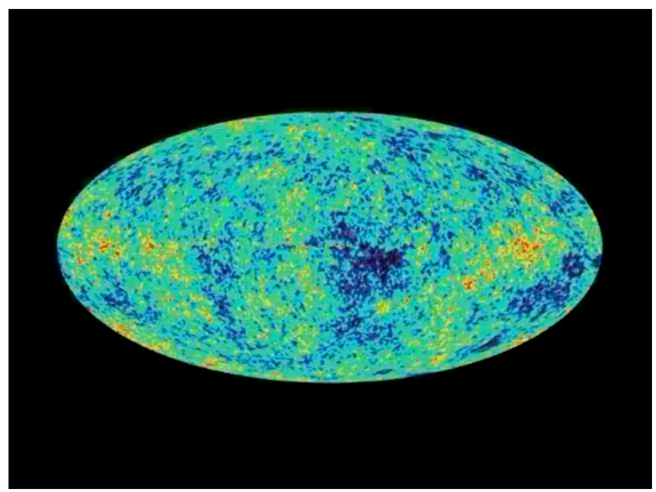
Quantum to Classical Perturbations

- Framework: quantum field theory in curved (quasi-de Sitter) classical spacetime
- Inflaton and graviton fields undergo vacuum fluctuations
- "Frozen in" to the classical metric at the horizon scale H, when a typical fluctuating patch has $E=hH$
- Continuous random noise: *infinite Hilbert space, infinite information*


time 2
time 1
quantum
fluctuations
inflation
"frozen"
fluctuation
size of
event horizon
size of
event horizon



electron wavefunctions in a hydrogen atom
(mathematical model)



inflaton wavefunctions in the early universe
(real data)

New Physics from CBR anisotropy

- Properties of inflaton field, potential
- Super-Planckian effects: initial vacuum
- Stringy Effects in tensor/scalar spectra
- pixelation, discreteness or new correlations from the quantization of spacetime

how quantum fluctuations turn into classical perturbations

consider a classical plane wave field mode

k/a = physical wavenumber

$u = a\delta\phi$ = conformal amplitude

$a \propto e^{Ht}$ = cosmic scale factor

$\tau = -(aH)^{-1}$ = conformal time

$$d\tau = dt/a$$

classical equation of motion (in conformal time)

$$\partial^2 u/\partial\tau^2 + \omega(\tau)^2 u = 0.$$

harmonic oscillator with varying frequency

$$\omega^2 = k^2 - 2(aH)^2$$

usual Born-Heisenberg-Jordan field quantization:

$$\mathbf{u} = w\mathbf{a} + w^*\mathbf{a}^\dagger$$

approximate solution for coefficients:

$$w = (2k^3)^{-1/2}(i - k\tau)\tau^{-1}\exp(-ik\tau)$$

solution at late times, field initially in vacuum state:

$$|w|^2 \;=\; a^2 H^2 (2k^3)^{-1}$$

yields spectral density of physical fluctuations:

$$P_\phi = (H/2\pi)^2$$

Dirac quantization: start with classical Hamiltonian

$$H_c = \frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2 + \frac{1}{2}\frac{\omega^2}{a^2}u^2$$

leading to Schrödinger equation for wavefunction:

$$-i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial u^2} + \frac{1}{2}[(k/a)^2 - 2H^2]u^2\psi$$

at late times, Schrödinger equation becomes

$$i\frac{\partial\psi}{\partial t} \approx H^2u^2\psi$$

solution is superposition of amplitude eigenstates

$$\psi(u) = \sum_j a_j \delta(u-u_j)\psi(u_j)$$

$$\psi(u_j) = \exp[-iH^2u_j^2 t]$$

with Gaussian distribution of coefficients amplitude 1= “live cat”

amplitude 2= “dead cat”

wavefunction includes live and dead cats

observers see only their own cats

observables are encoded in correlations

information in correlations is frozen in during inflation

information is subject to holographic entropy bound
at that time

Quantum Gravity

- Full theory: quantization of spacetime
- Candidates: loop quantum gravity, M theory
- Discrete elements, limited information
- Could quantum gravitational discreteness appear in CBR anisotropy?
- Quantitative result: holographic entropy bounds

The Holographic Principle

- *Regard black holes as coherent quantum objects* (unitary Schrödinger evolution)
- complete entropy $S=A/4$, where A is the area of the 2D event horizon in Planck units
- Includes quantum gravitational degrees of freedom (e.g., AdS/CFT, extremal black holes)
- This may generalize to a bound on the information content of any physical system

Susskind, ‘t Hooft, Bousso

Hawking-Bekenstein temperature of a black hole of mass M_{BH}

$$kT_H = \frac{\hbar c^3}{8\pi G\, M_{\mathrm{BH}}}$$

black hole entropy from unitarity

G. 't Hooft, hep-th/0003004

Transition probability for emitting particle of momentum *E*, in a given quantum state in volume *V*:

$$W\,\mathrm{d}t = \frac{\sigma(\mathbf{k})v}{V}\,e^{-E/kT}\,\mathrm{d}t$$

Cross section for absorption at high energies:

$$\sigma \approx 2\pi R^2 = 8\pi M^2$$

Assume Schrödinger equation governs entire system, then there are transition amplitudes

$$\mathcal{T}_{\mathrm{in}} = {}_{\mathrm{BH}}\langle M + E|\mathcal{T}|M\rangle_{\mathrm{BH}}|E\rangle_{\mathrm{in}},$$

$$\mathcal{T}_{\mathrm{out}} = {}_{\mathrm{BH}}\langle M|\,{}_{\mathrm{out}}\langle E|\mathcal{T}|M + E\rangle_{\mathrm{BH}}$$

Related to cross section and emission probabilities by:

$$\sigma = |\mathcal{T}_{\mathrm{in}}|^2 \varrho(M+E)/v$$

$$W = |\mathcal{T}_{\mathrm{out}}|^2 \varrho(M)\frac{1}{V}\,.$$

Where □(□) is the level density of states of a black hole of mass *M* time reversal invariance equates T_{in} and T_{out} , then divide probabilities to get

$$\frac{\varrho(M+E)}{\varrho(M)} = e^{E/kT} = e^{8\pi ME}$$

Integrate to get:

$$\varrho(M) = e^{4\pi M^2 + C} = e^{S}$$

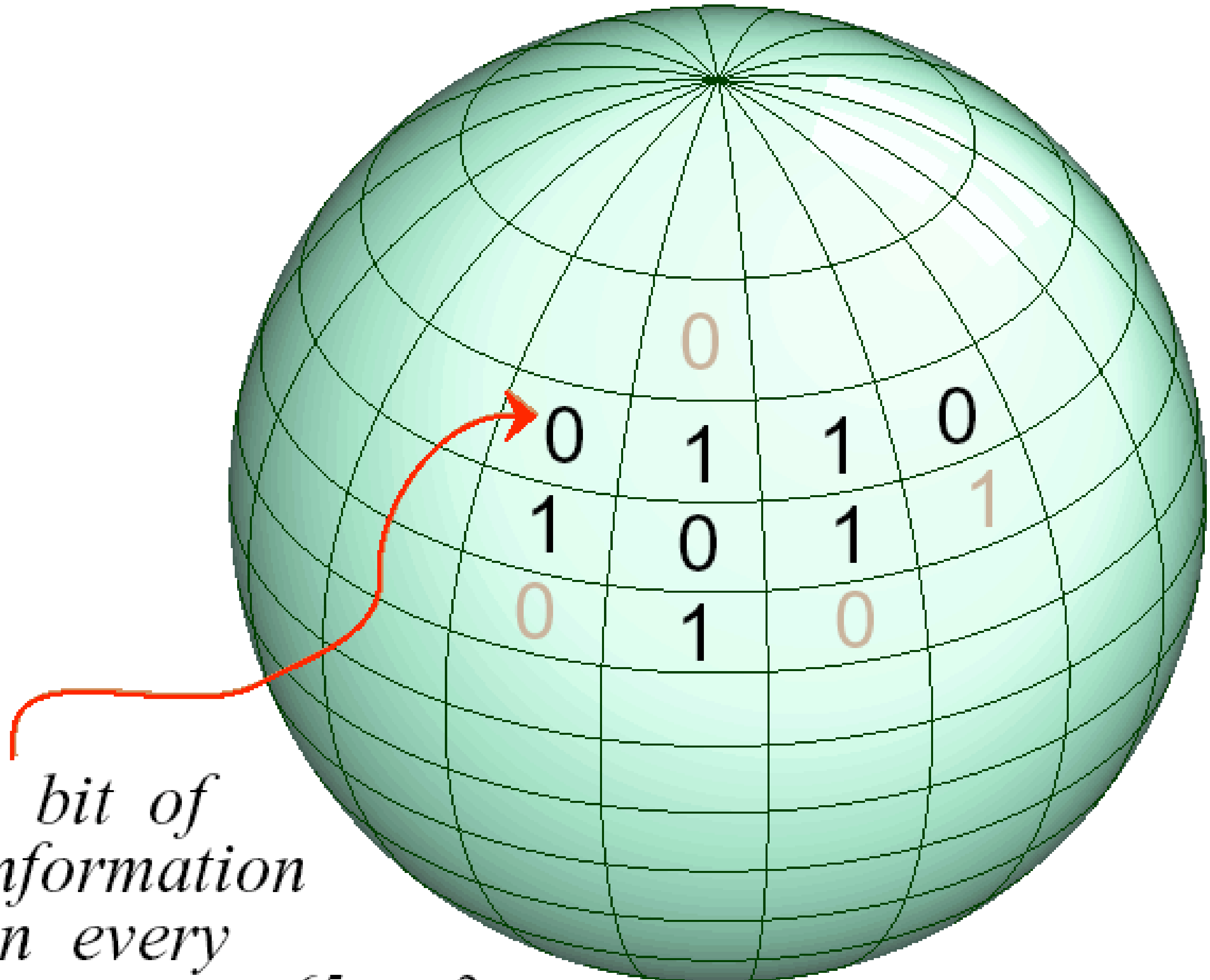
So the total number of quantum-mechanical states of a black hole, including spacetime states, is given by

$$\varrho(M) = 2^{A/A_0}$$

where

$$A_0 = 4 \ln 2 \, L_{\mathrm{Planck}}^2$$

is a fundamental unit of area



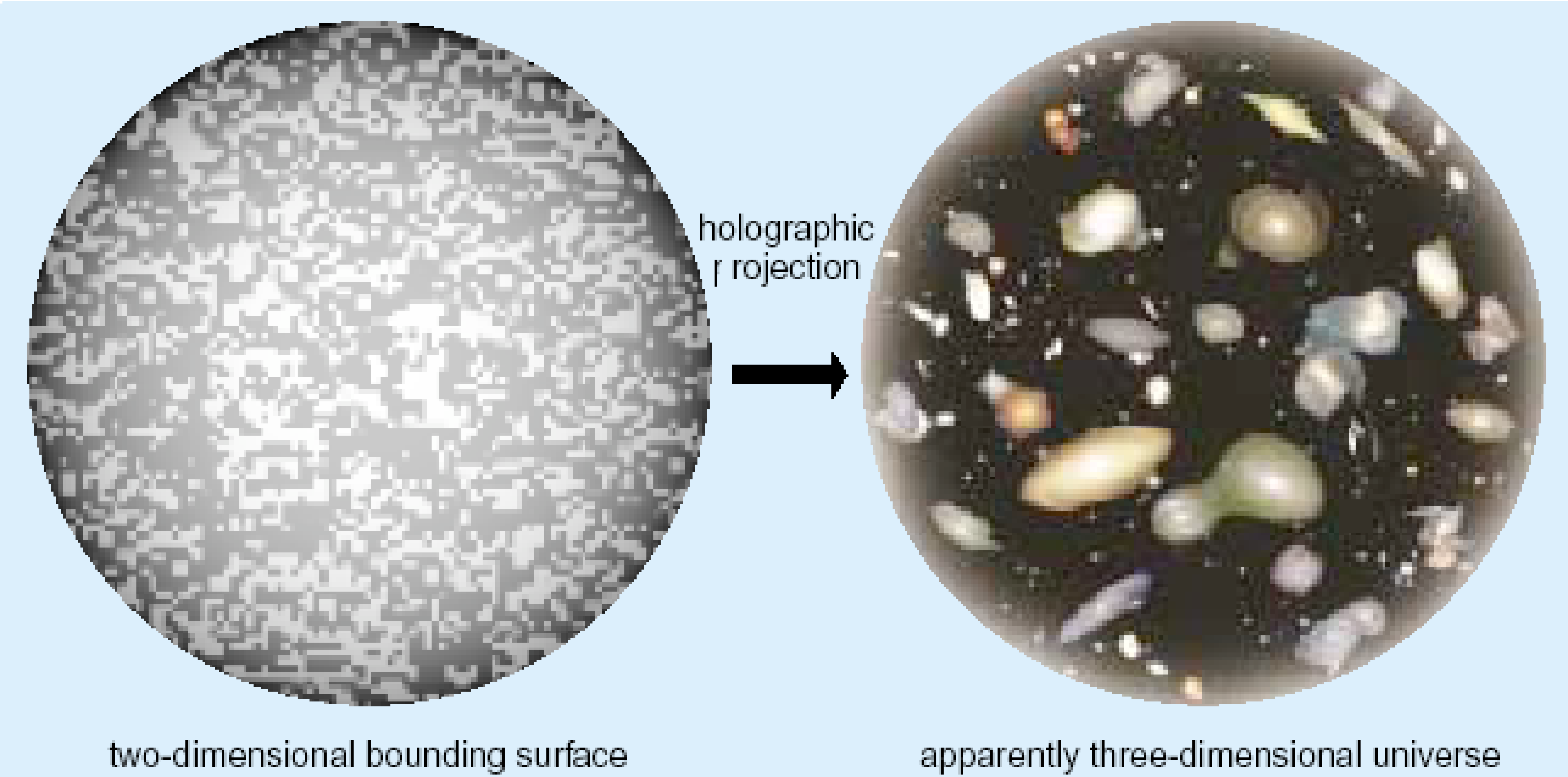
1 bit of information on every
0.724×10^{-65} cm^2

G. 't Hooft, hep-th/0003004

“This is what we found out about Nature’s book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.”

-Gerard ‘t Hooft

(Example of explicit holography in M theory: AdS5/CFT correspondence)


holographic
projection
two-dimensional bounding surface
apparently three-dimensional universe

Covariant Entropy Bound

- Black hole is most compact/highest entropy causally connected state of given energy
- The entropy of a 3D region [any "light sheet" volume] is bounded by one quarter of the area *A* of the 2D bounding surface in Planck units
- No exceptions yet found: fundamental structure?

Review: R. Bousso hep-th/0203101

Radicalness of Holography

- Hilbert space is *finite* and *discrete*: complete quantum state specified by

n=A/4 ln 2 qubits--- same information

content (2^n outcomes) as n binary spins
- Field theory, even single harmonic oscillator, has infinite Hilbert space
- Discreteness and nonlocality exist in nature, that are not modeled in field theory

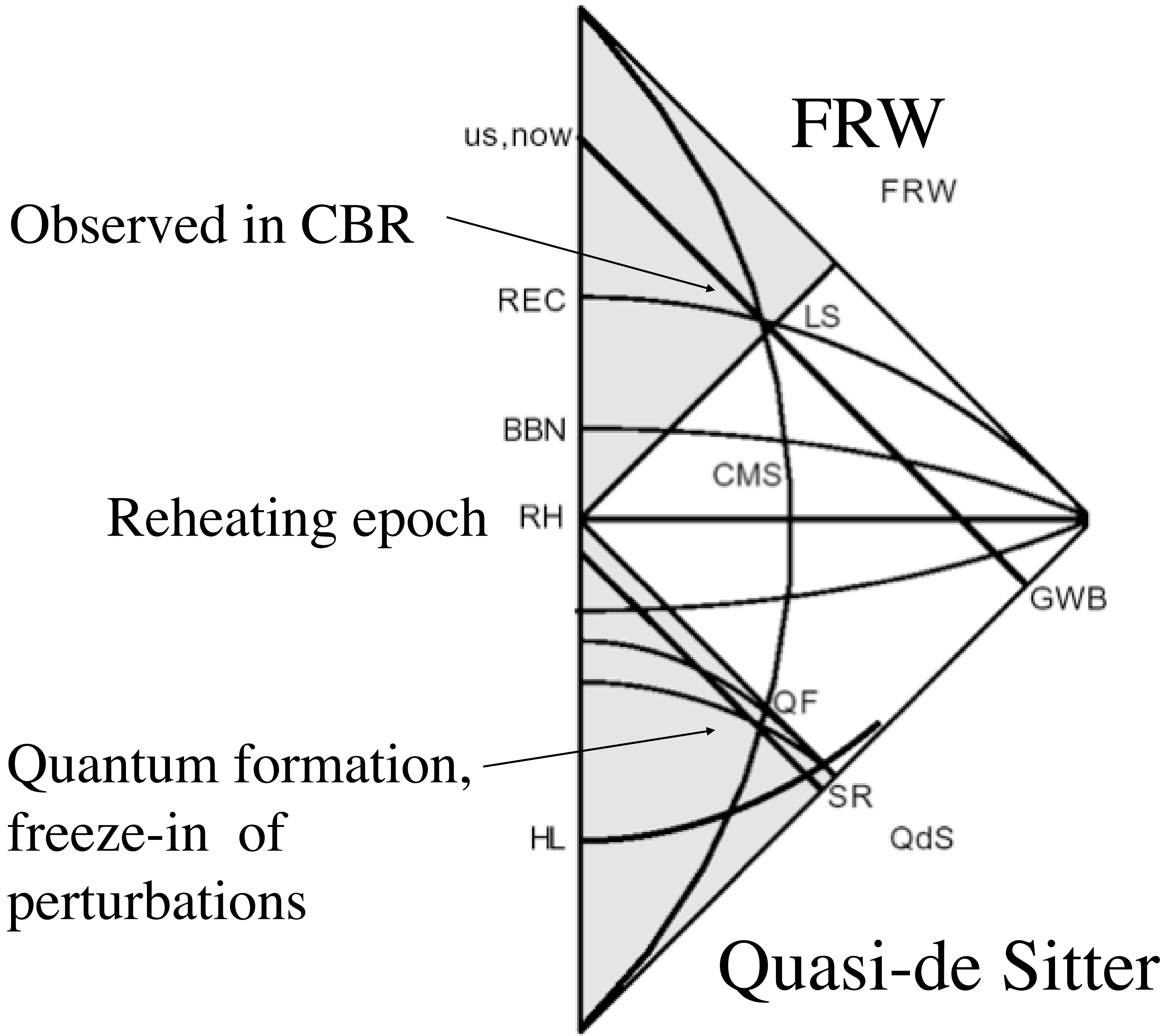
Qubits

- System of n entangled binary spins carries out a Hamiltonian quantum computation on n qubits

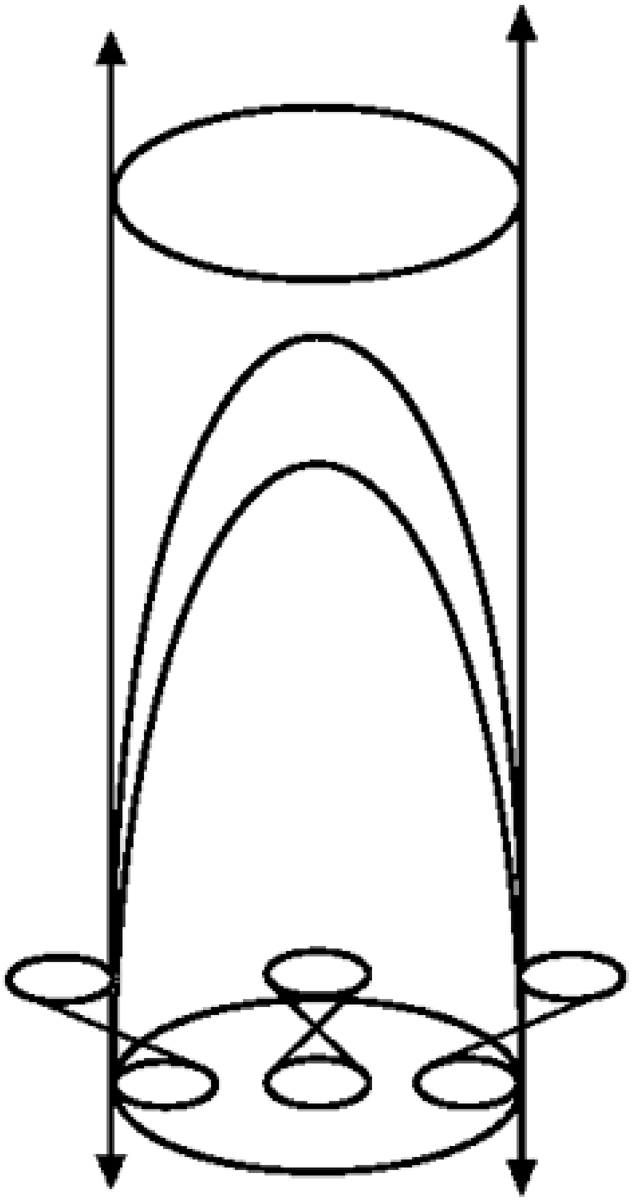
- When wavefunction “collapses” or “squeezes” into classical or pseudoclassical state, the final result contains n bits of information (it is one of 2^n binary numbers)

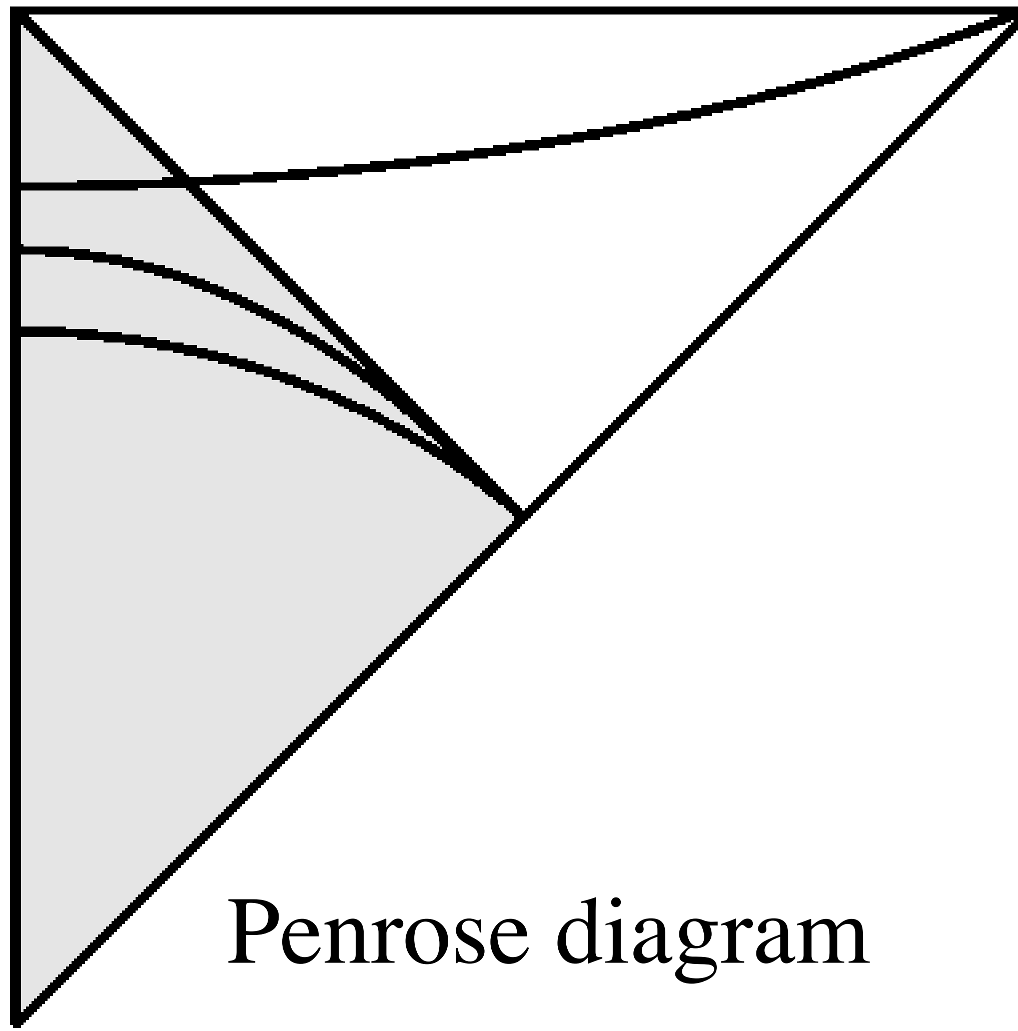
Holography and Cosmology

- Observable universe today is completely specified by 3□/□ ln 2= 10^{120} qubits
- Observable inflaton fluctuations may contain less information than this display


FRW
us,now
FRW
Observed in CBR
REC
LS
BBN
CMS
Reheating epoch
RH
GWB
QF
Quantum formation,
freeze-in of
perturbations
SR
HL
QdS
Quasi-de Sitter

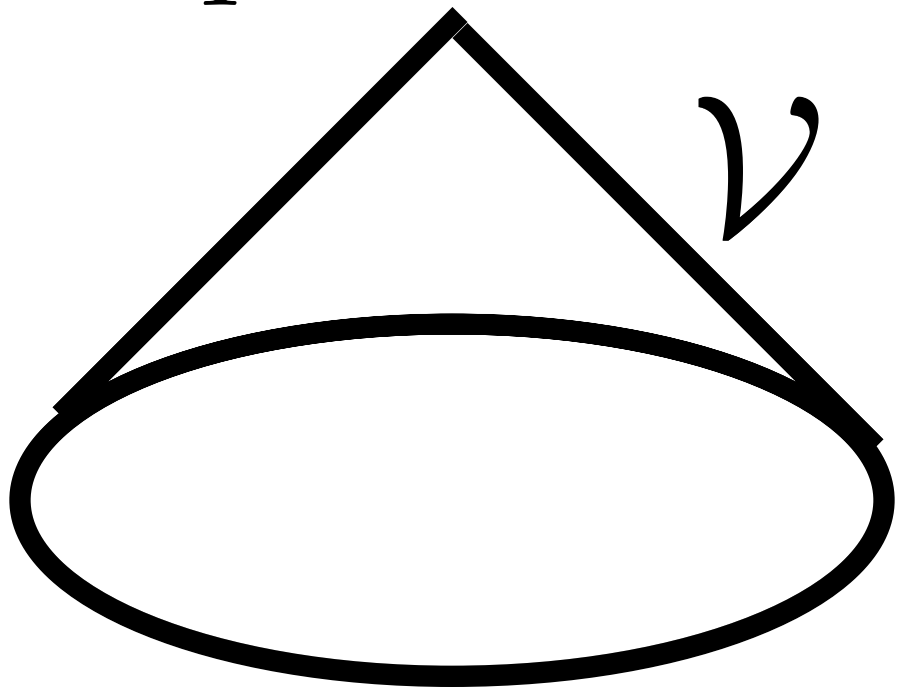
de Sitter Spacetime: physical and causal structure



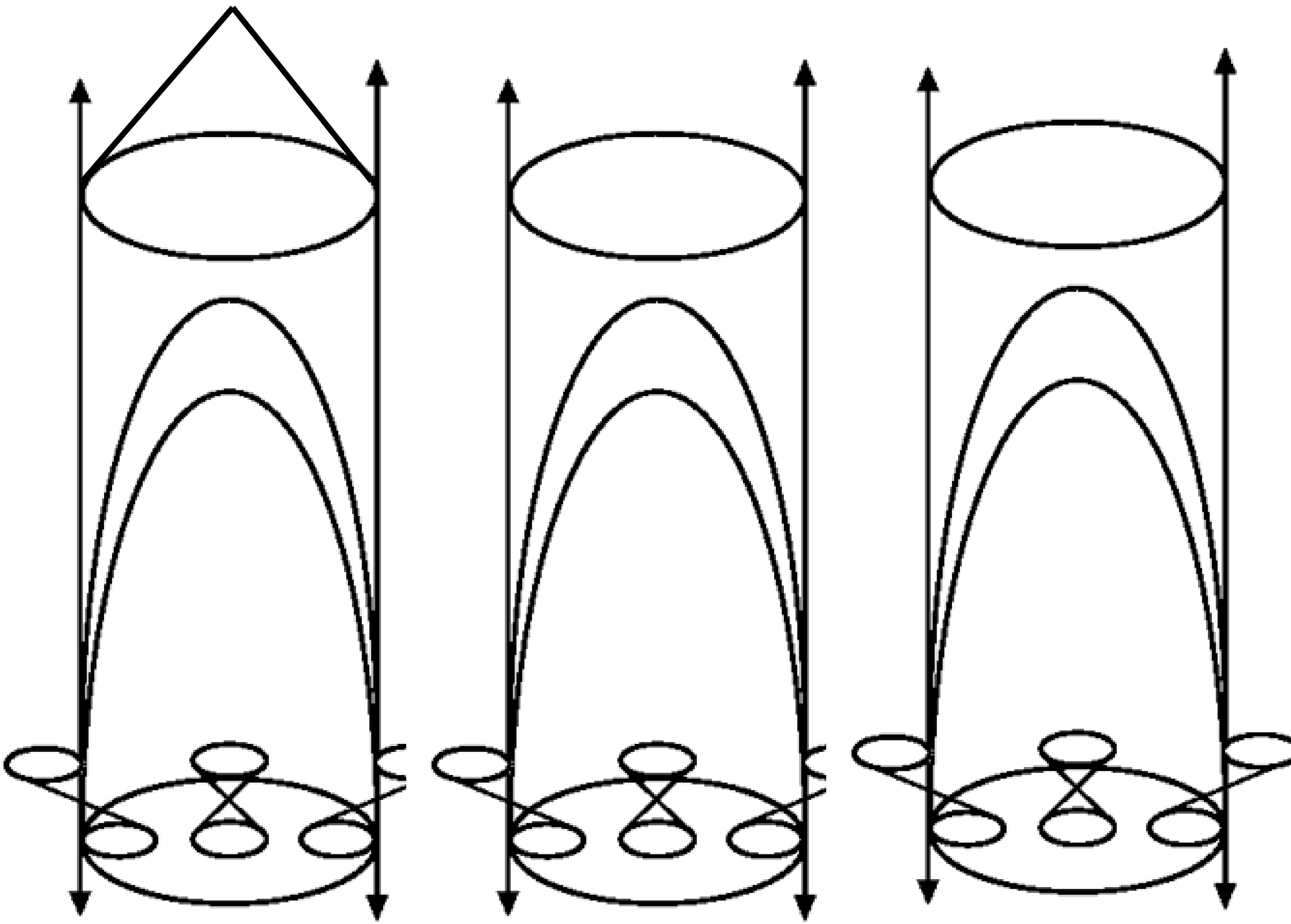


Penrose diagram

$$S_\mathcal{V} < \pi m_P^2 H^{-2}$$



$$S(V) < (V/V_H)\pi m_P^2 H^{-2}$$



mean frozen information in inflaton modes at

$$k/a < H$$

represents a fraction

$$f_H < 1$$

of the covariant bound on entropy density entropy of modes in a volume V_k of k space

$$n(k)V_k[V/a^3]I < (V/V_H)\pi m_P^2 H^{-2}$$

leads to maximum density, or minimum mean separation of independent modes

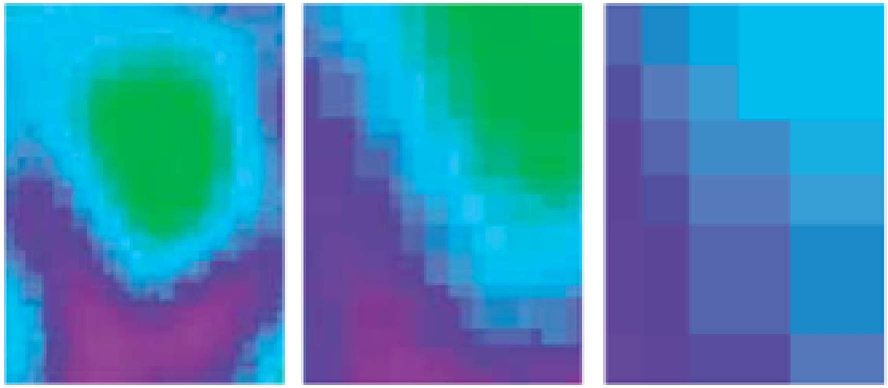
$$(\Delta k/k) > I_H^{1/3} D^{1/3}$$

$$D \equiv \left[\frac{4}{3}\frac{H^2}{f_H m_P^2}\right]$$

parameters of inflation and quantum gravity;

10^{-10} ? 1?

example: pixelated k space



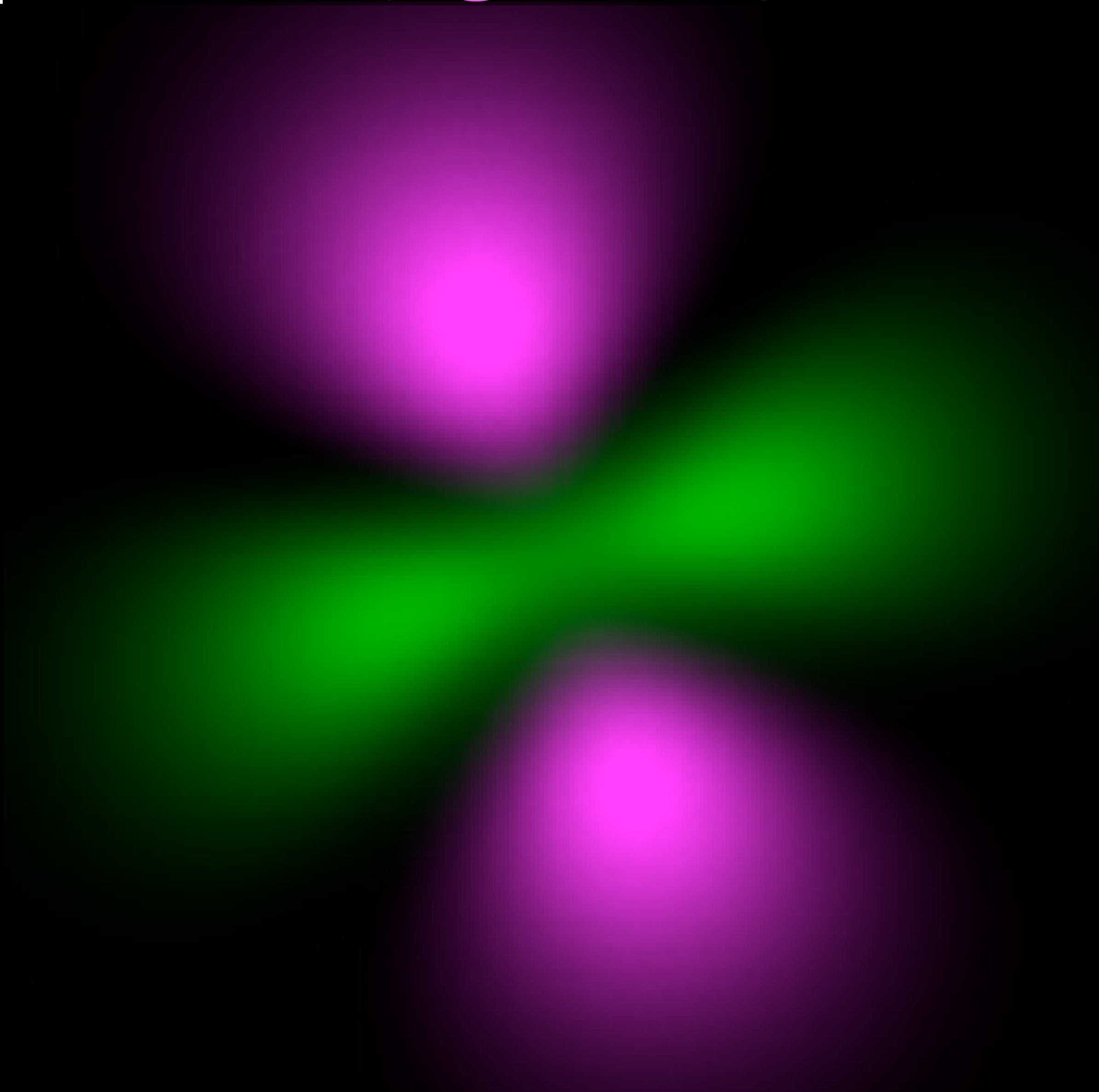
coherence angle and modulus width of pixels obey

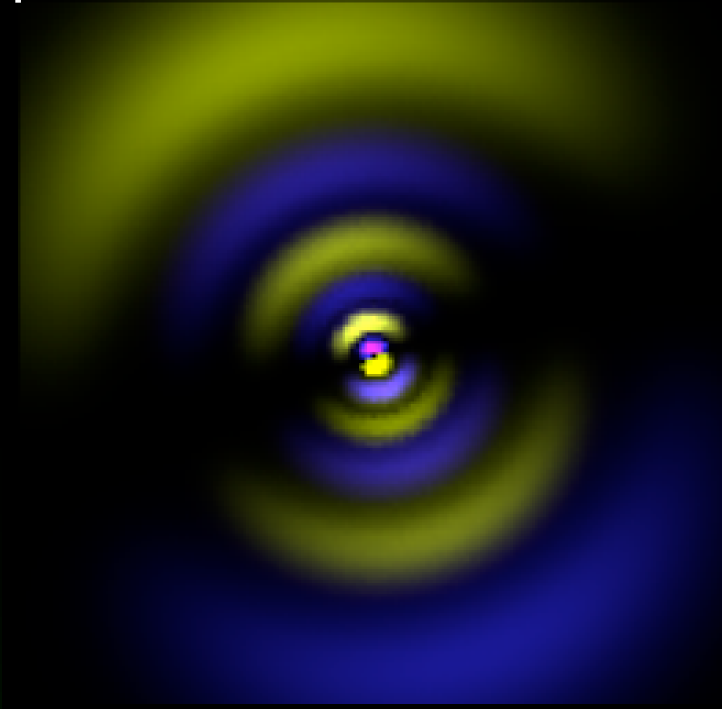
$$\delta\theta^2(\delta k/k) > I_H D$$

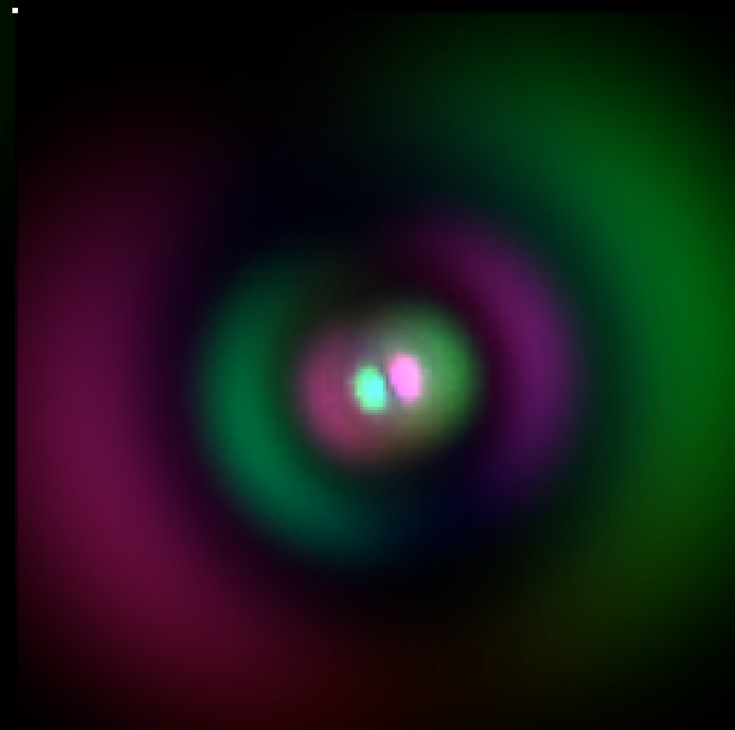
Observable discreteness?

- Each horizon-scale inflaton when it freezes out carries less information than 10^{10} binary spins
- Encoding is unknown (depends on quantum gravity)

H atoms

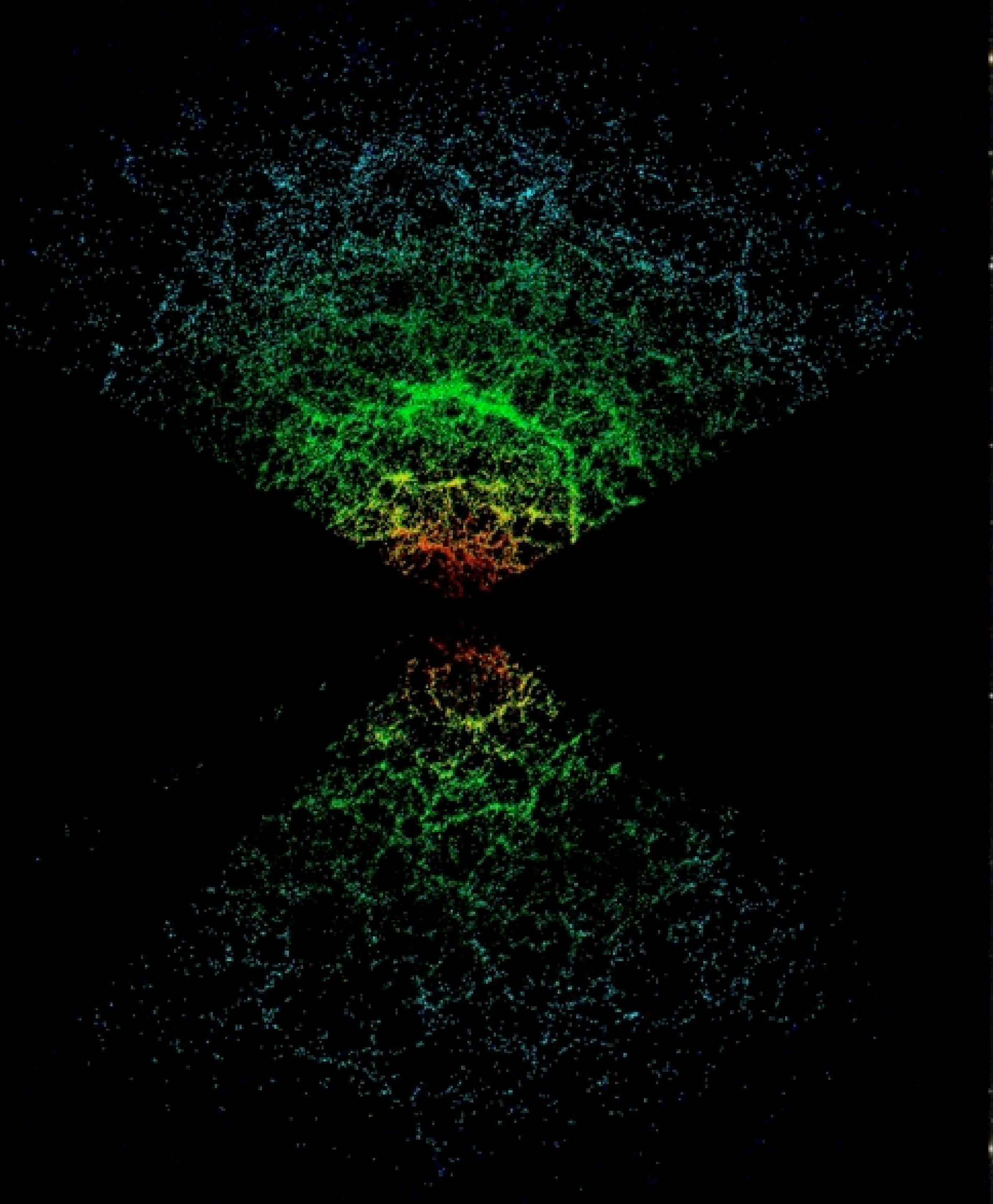


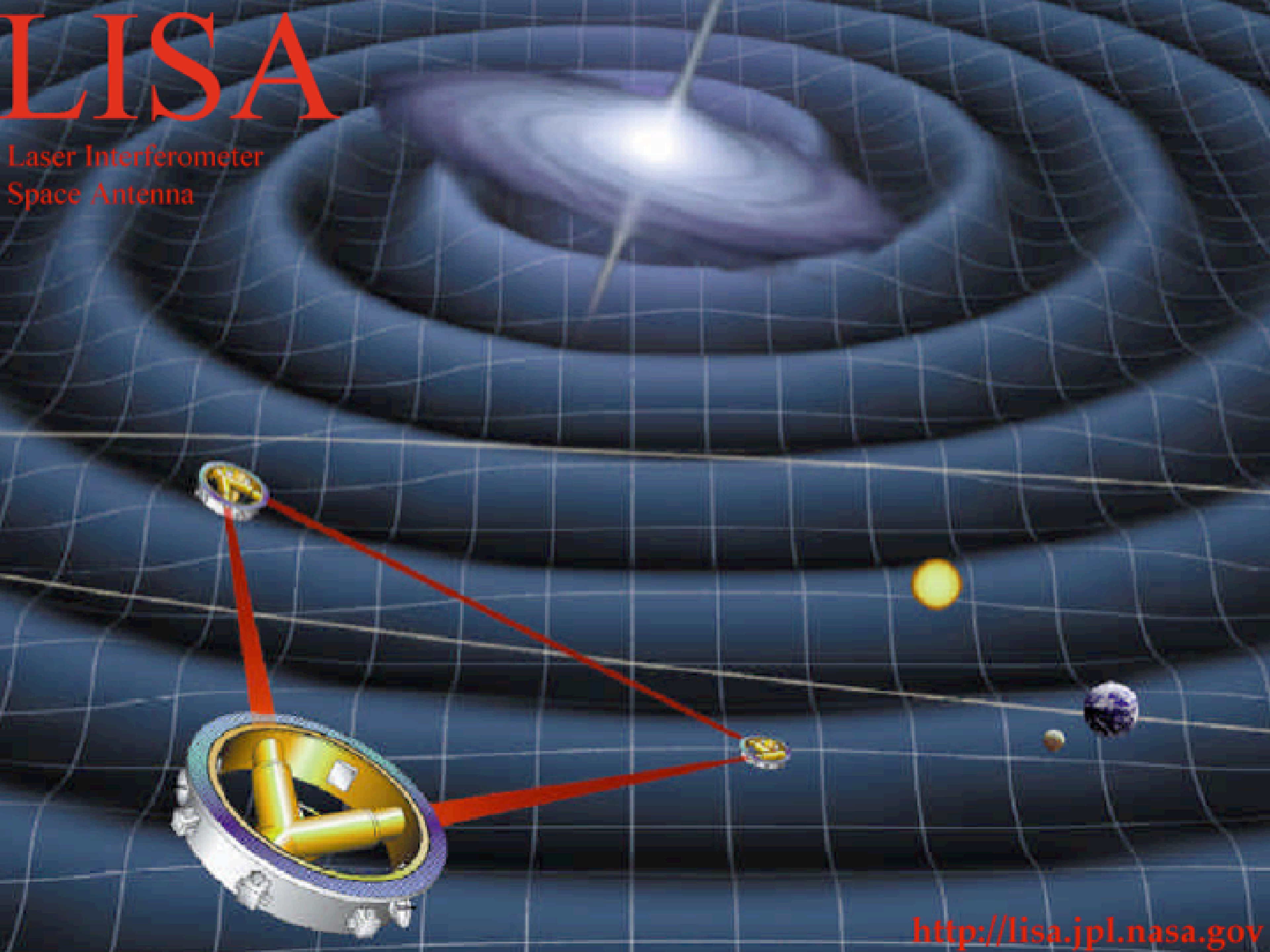




Real World

- Observability depends on nature of the quantum-gravitational eigenstates
- *Not* observable: discreteness corresponding to increments of temperature, $\square T/T=10^{-10}$ (eigenstates covering large solid angles)
- *Possibly* observable: new correlations, discreteness or “pixelation”
- Real quantum gravity…….?




LISA
Laser Interferometer
Space Antenna
http://lisa.jpl.nasa.gov